

Mechanisms for Human Spatial Competence

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Abstract. Research spanning decades has generated a long list of phenomena associated with human spatial information processing. Additionally, a number of theories have been proposed about the representation, organization and processing of spatial information by humans. This paper presents a broad account of human spatial competence, integrated with the ACT-R cognitive architecture. Using a cognitive architecture grounds the research in a validated theory of human cognition, enhancing the plausibility of the overall account. This work posits a close link of aspects of spatial information processing to vision and motor planning, and integrates theoretical perspectives that have been proposed over the history of research in this area. In addition, the account is supported by evidence from neuropsychological investigations of human spatial ability. The mechanisms provide a means of accounting for a broad range of phenomena described in the experimental literature.

Keywords: Spatial Cognition, Cognitive Architecture, Computational Model, Frame of Reference, Vision, Representation, Mechanism, ACT-R.

1 Introduction

In this paper, we present a broad theoretical architecture for understanding human spatial competence. Human spatial abilities are brought to bear in a variety of contexts, and in a variety of ways. Spatial information processing is utilized for navigation and wayfinding [1], [2], map reading and orientation [3], [4], [5], and spatial transformations like mental rotation [6], [7]. However, spatial abilities are also recruited for syllogistic reasoning tasks [8], problem solving [9], [10], and language processing [11], [12]. This flexibility and diversity requires that an account of human spatial abilities be able to address a range of specific abilities within the context of overall cognitive functioning.

In addition to breadth, an understanding of human spatial competence requires a grasp of the details of the mechanisms involved in encoding, processing, and using spatial knowledge. This includes questions concerning how spatial information is represented, as well as the mechanisms that are available for manipulating those representations [13], [14], [15]. The literature contains many theories that address various aspects of spatial information processing, including representations of environmental information [11],

[16], [17], visuospatial working memory [18], [19], reasoning with spatial mental models [20], [21], mental imagery [14], [22], and navigation [23], [24], [25]. What currently does not exist, however, is an integrated theory that provides an account of human performance across different domain areas.

The theory presented in this paper addresses each of these general areas of human spatial competence, to provide broad coverage on how humans encode, store, and use spatially-based information to perform a variety of tasks in different domains. Because of the scope of the challenge, we have tried to strike a balance between presenting the breadth of the theory, while describing the components in sufficient detail to permit a thorough evaluation. We have grounded the account in the ACT-R cognitive architecture [26], which provides a well-validated theory of overall human information processing. We do this to connect our work to a more general theory of human cognition. This provides us with important constraints on our account and allows us to focus more specifically on mechanisms for spatial information processing, since the existing ACT-R architecture provides validated mechanisms for other critical components of the human cognitive system. Although the mechanisms we propose are not implemented yet, they are specified in enough detail to identify accounts for various phenomena, some of which are described briefly in the remainder of this chapter. To begin, we address several important issues in the realm of spatial competence in the next several subsections. Dealing with critical concepts from the literature at the outset hopefully will clarify our approach and simplify the discussion of other points in the remainder of the paper.

1.1 The Cognitive Map

Tolman's seminal article, "Cognitive Maps in Rats and Men" [27], is generally associated with the origin of modern research into spatial information processing. Since then, the term *cognitive map* has played a central role in theorizing about human spatial abilities. Many theories have been developed that claim humans automatically generate an exocentric cognitive map of the environment based upon experience in a space (c.f. [28], [29], [30]). Proponents of these theories have pointed to the discovery of place cells in the rat [31] and human [32] hippocampus as key evidence for this view. The alternative that is most commonly offered is egocentric encoding of spatial information, where the locations of items in the environment are encoded with respect to the coordinate system defined by the location and orientation of the viewer (e.g., [11], [33]).

Evidence has accumulated on both sides of this debate (e.g., [34], [35], [36], [37]). However, we find the evidence arguing against the exocentric cognitive maps as the default representational format for human spatial representations to be compelling. This is not to say that humans can not or do not sometimes represent space using exocentric reference frames. Rather, our claim is that humans do not *automatically* construct a *cognitive map*¹ of the environment based on visual perception. Instead, we believe that spatial information is encoded in a fragmented manner by default, using

¹ We use the term 'cognitive map' to refer to the notion of an internal, exocentric representation of space that is akin to a paper-based map. While the term initially held a much broader connotation, this has been largely lost in current usage.

multiple coordinate systems to represent spatial locations. Initially, early vision utilizes a retinotopic coordinate system, which can be used for guiding and directing visual attention [38]. We propose that the perceptual system generates two enduring, high-level encodings of spatial location from visual input, one based on the egocentric frame of reference (distance & bearing from self), and one based on a frame of reference defined by salient features of the environment (e.g., the boundaries of a room or a prominent landmark). The evidence for these representations comes from functional considerations, described next, and findings from neuropsychological research (Section 4).

Importantly, egocentric and exocentric frames of reference support different functions within the system (e.g., [33]). Encoding location with respect to an egocentric frame of reference facilitates acting on objects in the world ([17], [39], [40]). To interact with an object, it is critical to have knowledge of the relationship between oneself and the object. In addition, this representation of location is a primitive in visual perception, where perceived distance and bearing of an object can be inferred directly from the visual stimulus [33]. In contrast, location information based upon an exocentric frame of reference is important for grounding spatial information in the environment and for computing spatial relations. For these tasks, it is necessary that locational information be represented within a common coordinate system. The egocentric reference frame is not appropriate for such tasks, since any movement or rotation by the viewer produces a change to the coordinate system [33]. Thus, location information based on an exocentric reference frame is needed to link locational information for multiple objects for making spatial judgments. Spatial processes, in conjunction with imagery, can be applied to generate more complex representations for multiple objects from these elements as well (e.g., a cognitive map). However, this is an *effortful* process that inherits the *error* and *bias* that is associated with human visual perception, not an automatic, unconscious process providing an integrated representation of the environment.

1.2 Hierarchical Encoding

There is substantial evidence for a hierarchical component to spatial information processing (e.g., [41], [42], [43]), and any serious theory of human spatial competence needs to account for these findings. In our account, hierarchical phenomena arise as a consequence of the frames of reference used for visual encoding. A frame of reference is used to encode visual information, based upon the contents of the visual experience. To take a famous example from Stevens & Coupe [43], when studying a map of the United States, San Diego will tend to be encoded with respect to the state of California, and Reno will tend to be encoded with respect to the state of Nevada. To compare the relative locations of these two cities, however, requires that they be positioned within the same frame of reference. In this case, it is necessary to shift to the United States as the frame of reference. The relative spatial locations of the two states within the United States will lead to the typical error (i.e., believing that Reno is farther east than San Diego, when it is actually farther west).

We are unable to provide a full discussion of the mechanisms that would support these operations in this paper. However, the key point with regard to hierarchical encoding is that each item encoded by the system is represented within an exocentric

reference frame based upon local, salient features of the environment. Hierarchical phenomena arise because that reference frame is then represented as an item in a larger reference frame. Thus, San Diego (the item) is positioned at a particular location within the state of California (the reference frame). However, California occupies a particular location within the United States. Our assumption that spatial comparisons must be carried out within the same reference frame provides the explanation for why various hierarchical phenomena are found in spatial tasks. Mentally re-encoding location relative to a new reference frame takes time and results in increased error and bias.

1.3 The Imagery Debate

Finally, mental imagery has generated a substantial amount of research and theorizing throughout the history of psychology [13], [14], [15], [22], [44], [45]. A major issue under debate has been whether visual mental images are depictive. That is, do mental images have a spatial extent (in the brain) that preserves the spatial properties of the original stimulus? More generally, the question concerns an issue of whether mental images are encoded in a format that is distinct from other kinds of information stored in the brain.

To resolve this issue, we look to the representations and mechanisms in the ACT-R architecture. ACT-R posits a number of processing modules, which are responsible for different aspects of cognition. In the architecture, there is a vision module, which is specialized for processing visual perceptual information. We agree with Kosslyn and others that mental imagery utilizes many of the same cortical areas and neural pathways as vision [22], [46]. Consequently, our theory tightly couples mechanisms for mental imagery with existing architectural mechanisms for visual perception. The result is that vision and mental imagery operate on the same representations, which are different from other information in declarative memory. It is interesting to note, however, that this distinction is based largely on content. All declarative knowledge in ACT-R, including visual chunks, is represented propositionally. Thus, while visual knowledge is distinct, the representation is not necessarily qualitatively different from other knowledge in memory. This speaks to the more detailed issue of whether visual mental images are depictive in a real sense. One reason for propositional representations of visual information in ACT-R is the architecture's relatively abstract and lean representation of visual information. However, it is also the case that propositional representations are more in line with the existing architecture. To the extent possible, we are working within the overall structure of the architecture, until evidence arises that forces us to rethink some of these assumptions. For now, we believe that the representation of visual information currently instantiated in ACT-R provides an adequate foundation that supports the additional representational components and mechanisms we intend to implement.

2 Unified Theories of Cognition and ACT-R

Cognitive architectures, like ACT-R, EPIC, and Soar, instantiate a theory of the human information processing system in its entirety. These unified theories of

cognition [47] contain mechanisms to account for various aspects of human cognitive functioning, including problem solving, perception, and motor actions [26], [47], [48]. One of the challenges associated with developing a cognitive architecture is identifying an appropriate set of mechanisms, which are not only capable of producing solutions to a broad range of tasks faced by humans, but which solve those tasks in a psychologically plausible manner. Because of the prevalence of spatial information processing in human cognition and performance, it is critical to incorporate mechanisms for spatial processing in these theories, particularly as cognitive architectures are applied to increasingly complex, spatially rich tasks. In addition, however, it is vital that theories of spatial competence take seriously the constraints imposed by other components of the human cognitive system, many of which have been implemented in cognitive architectures. Human perception and action is constrained in ways that can significantly influence performance on spatial tasks. In addition, human cognitive limitations, like working memory capacity and long-term memory decay moderate how spatial information is processed and remembered. In short, theories of human cognition cannot ignore spatial information processing, just as theories of spatial competence must take into account other perceptual, cognitive, and motor mechanisms.

For the most part, unfortunately, these research communities have remained disconnected. Our intent is to incorporate what is known about human spatial competence into a cognitive architecture to facilitate developing more precise, and psychologically valid, quantitative accounts of human performance on complex, spatially-demanding tasks. Researchers in the area of spatial cognition have developed a variety of theories to account for human performance in different spatial information processing domains (e.g., [19], [20], [22]). These theories capture important capacities and limitations of human spatial ability. However, they are often not implemented. And, when they are, they are typically not implemented as part of a more comprehensive theory of human cognition (e.g., [21], [49], [50]). In the remainder of this paper, we describe our proposal for linking the insights of this research to a sophisticated, yet general, computational theory of the human information processing architecture.

2.1 ACT-R

A full description of the ACT-R architecture is beyond the scope of this chapter. Thus, only a brief sketch is given here. More detailed descriptions can be found elsewhere (e.g., [26], [51]). ACT-R is a cognitive architecture with a set of core mechanisms that has been used to provide accounts of human performance across a broad range of research domains (see [51] for a review). At the highest level, ACT-R is a serial production system where productions (condition-action pairs) are matched against the current state of the system. On each cycle, a single production is selected and executed (fired), which produces a change in the state of the system, and the cycle begins again. The current state in ACT-R is defined by the contents of a set of buffers. Each buffer is associated with a specialized processing module, and serves as the interface between the module and the production system. We mentioned the vision module above, which has a buffer to represent object properties (*what*), and a second buffer to represent location information (*where*). There is also a declarative memory

module with a retrieval buffer, which is specialized for storing and processing declarative knowledge (facts and information stored as *chunks*). Each buffer may hold only a single chunk at any given time, and each module can process only a single request at a time. Thus, modules and buffers are serial as well. Parallelism exists in ACT-R through the simultaneous operation of all of the modules. Subsymbolic mechanisms are implemented within the modules and produce a graded quality in cognitive processes. The speed and accuracy of operations are impacted by continuously varying quantities, like activation for declarative knowledge and utility values for productions.

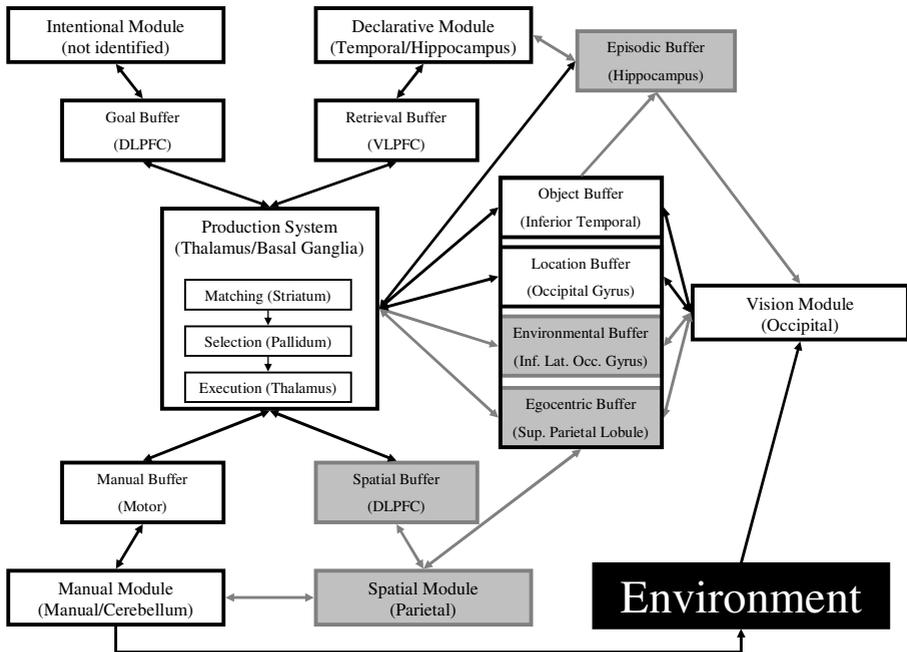


Fig. 1. Schematic illustration of the current ACT-R architecture, with proposed additions included. Structures identified in white represent existing components of the architecture. Grey components represent proposed additions. The *environment* is indicated in black.

The modules generally are driven by requests from the production system. For instance, a production may request a shift of visual attention. The module processes the request and returns the result to the buffer, where it can be accessed by the production system. In the case of shifting attention, the vision module plans and executes the action, and a chunk representing the item being attended is placed into the visual-object buffer. Figure 1 illustrates the major components of the current ACT-R architecture, along with the additions that are proposed in this paper. The current version of ACT-R (ACT-R 6.0) has been designed and implemented to support adding, modifying, or deleting components, out of an appreciation of the limitations of the current architecture and interest in having research explore

alternative accounts of cognitive phenomena [26]. This makes ACT-R well-suited for exploring how to account for human spatial competence in the context of a unified theory of cognition. In the next section, we describe our suggested modifications and additions, including how they integrate and interact with the existing architecture.

3 An Architectural View of Spatial Competence

Our account of spatial competence in ACT-R consists of proposals to add a module and several buffers to the architecture, in conjunction with mechanisms to support the kinds of processing performed by humans on spatial information. In general, this proposal is in line with the existing architecture, and with existing practice within the ACT-R community. One exception to this is an explicit proposal for direct communication between modules. Although such connections do not exist in the architecture currently, there is a recognition that they are likely to exist, based both on human performance and neuroanatomy (Anderson, personal communication). We propose a close link between the new spatial module and other modules in the architecture, particularly the vision and motor modules. Overall, we have taken care to ensure that the proposal is consistent, both internally and with ACT-R. Thus, we are confident that the emerging account provides a useful conceptualization of how humans encode, store, and process spatial information.

3.1 Enhanced Visual Representation

The existing representation of visual information in ACT-R is based substantially on the EPIC architecture [48]. It represents visual information by splitting *object* information from *location* information, following the research of Ungerleider & Mishkin [52]. However, these representations are impoverished, due to both historical and technical reasons. Cognitive architectures certainly have not solved the vision problem, nor does our theory. However, we propose to augment the existing representation of visual information, specifically location information, to provide a more psychologically valid representation that is able to support spatial operations.

The basic functioning of the vision module in ACT-R is that the contents of the screen are processed into the *visual icon*, which is a transient representation in a retinotopic frame of reference (actually, locations are based on screen coordinates out of convenience), which is similar to a feature map [53]. Although the ACT-R visual icon is not depictive in the sense that Kosslyn's [22] visual buffer is, we propose that the icon serves similar functions with regard to the construction and use of visual imagery (Section 3.2). Shifts of attention in ACT-R occur when a production includes a request for an attention shift, which specifies constraints on where attention should go. These constraints can be based on *location* (e.g., to the right of where attention is currently), and *features* of the objects displayed (e.g., only blue objects). The constraints are compared to the information available in the icon, and the items that match those constraints are identified. One of the items matching the request is selected (randomly if there are multiple items that match), and attention is shifted to the new location. Once attention "arrives" the visual buffers are populated with

chunks representing information about the object. The timing of these operations is based on a vast psychophysical literature.

Egocentric Buffer. The first step in augmenting the representation of location in ACT-R consists of adding a buffer to hold a representation of the egocentric location of the object (the egocentric buffer in Figure 1). Currently, ACT-R's visual-location buffer holds the location of the object using screen-based coordinates. This buffer includes other featural information about the object, including its size, color, and type (e.g., *text* versus *button*), which corresponds roughly to the information available in the visual icon. In practice, this representation is used primarily to support visual search, but it also supports processing of 2D displays, as are commonly used in psychological experiments.

What the existing representation does not support is encoding location in 3D space. Historically, this has not been problematic, since ACT-R (like other cognitive architectures) generally has not been applied to tasks involving complex, 3D environments. This kind of task environment, however, is becoming increasingly common as the applications of work on computational cognitive modeling continue to expand (e.g., [54], [55], [56]). To address this shortcoming, we propose adding an egocentric buffer to hold 3D spatial information. Information encoded in this buffer includes the distance of an object, as well as its bearing, relative to the location of ACT-R in the environment. It also includes an estimate of the absolute size of the object (i.e., not retinal size), as well as the orientation of the object and information about motion (speed and direction). Like existing buffers in the vision module, this information is encoded and updated when visual attention is shifted to a particular object. Note that the existing visual-location buffer remains essential. We believe that visual information represented at the level of features in a retinotopic frame of reference is necessary in the control of visual attention.

Environmental Frame of Reference Buffer. As important as an egocentric encoding of location is for immediate action and processing, it does not provide any information about the location of an object relative to other objects in the world. In Section 1.1, we indicated that representing location information utilizes multiple frames of reference. One of these is based upon the surrounding environment. In virtually any space, there are distinct features that provide a frame of reference for encoding relative locations of objects. This may be a landmark, like the Eiffel Tower in Paris, or geographic feature, like the Pacific Ocean on the California coast in the United States. We propose that the human visual system takes advantage of these features to provide a stable frame of reference for encoding object location. An interesting question exists regarding how a particular reference frame is selected within an environment when multiple options are generally available. We believe that salience plays a key role in this process. However, we suspect that there are large individual differences in this process, which may contribute to performance differences in orientation and navigation tasks [57], [58].

The contents of the environmental buffer provide a basis for calculating spatial relationships among objects. Some proposals suggest that these quantities are computed automatically when visual attention is shifted from one object to another (e.g., [59]). In contrast, we believe that identifying the spatial relationship between

two objects is an explicit process of estimation. However, if objects are represented in an exocentric frame of reference, such estimates need not be difficult to compute, and can be determined using immediate visual perception or from memory using mental imagery. So, while you may not have explicitly considered the distance between the phone in your office and the door, you can recall the locations of both items (within the reference frame of your office) from memory and compute that relationship with relative ease when asked. If items are located in different frames of references (e.g., the stove in your kitchen and the computer in your office), additional effort is needed to establish a common frame of reference, but the process will be similar. Mental imagery, described below, supports these operations.

Episodic Buffer. The final modification we propose to the vision module provides a means for consolidating visual experiences into a unitary representation. We accomplish this by proposing an episodic buffer, which links the contents of the visual buffers, and produces an episodic trace of the experience. Our proposal bears significant resemblance to Kosslyn’s conceptualization of the role of the hippocampus in representing episodic information [22]. Specifically, we do not propose that all of the information related to a visual experience is represented in this buffer. Rather, we propose that this buffer holds a chunk that encodes pointers to the contents of the other visual buffers (other chunks).

The resulting vision module should operate as follows. When attention is shifted to a new object in the visual field, the vision module updates the visual-object, visual-location, environmental, and egocentric buffers with chunks that represent the information about the object being attended. These processes occur in parallel, through distinct mechanisms in ACT-R, and distinct cortical pathways in the brain (see Section 4). Identifiers for those chunks are specified as slot values in a chunk in the episodic buffer, linking them together in a single episodic representation. All of these chunks are deposited into declarative memory, making the information accessible at later times. In addition, these chunks also are subject to the same activation learning and decay mechanisms as other chunks in memory, meaning that perceptual experience can be forgotten much like other information. These mechanisms already exist in ACT-R. The chunks stored in memory form the basis for mental imagery, which is discussed next.

3.2 Mental Imagery

There is a great deal of evidence suggesting that engaging in mental imagery recruits many of the same areas of the brain as visual perception (c.f., [22]). Based on this literature, we find it appropriate to posit a close link between visual perception and imagery. In fact, our claim, in line with Kosslyn, is that mental imagery does not reflect a distinct and separable component of human cognition. Rather, in this architecture, mental imagery operates through the mechanisms associated with visual perception. Note that in Figure 1, there is no imagery module and no imaginal buffer, in contrast with Anderson et al. [26]. We achieve the functionality associated with mental imagery through the interaction of the vision module with a spatial module that incorporates default features of modules within ACT-R.

Images are generated in this architecture by retrieving episodic perceptual experiences from memory. This process works similarly to retrieving declarative knowledge in ACT-R, such that a request is generated by the central production system to retrieve an episodic chunk from memory, which is placed in the episodic buffer, with pointers to the chunks associated with this memory from the other visual buffers. These chunks can be retrieved based on these references and the information can be propagated to the visual icon, where it is reinstated. This process causes attention to be “pulled from” the external environment and focused on this internally-generated visual representation. The result is, essentially, a copy of the original visual experience. However, a variety of errors could occur in this process, including mis-retrievals, which would affect the characteristics of the mental image.

As the preceding description suggests, we posit that mental images are represented at the level of the visual icon in ACT-R. In the current implementation of ACT-R, this is a propositional representation that contains feature-based information about objects, including spatial location, color, and size [51]. Because mental images are effortful to maintain, we posit that the visual icon has a rapid decay rate. It is only by refreshing information that it can be maintained. For items in the visual world, this is effortless, since the electromagnetic radiation impinging on the retina provides constant input regarding visual information in the environment. In the case of mental images, however, attention is required to maintain the image for any significant length of time. As we implement this architecture, we will adapt ACT-R’s existing declarative memory decay function for use with this component of the system, with an appropriately higher decay rate.

Of course, mental images can be modified and transformed. The mechanisms available for performing these transformations are described next. We simply note here that the primitive transformations available for manipulating mental images relate to slot values in the chunks created during visual perception. Whereas a variety of transformations are possible, we focus in the next section on *spatial* transformations, which are generated through changes to slots in the chunk in the egocentric buffer.

3.3 A Specialized Module for Processing Spatial (and Magnitude) Information

Spatial information processing is a component of many tasks and cognitive activities. Thus, an account of these abilities in humans must be both general and powerful. The modifications to the vision module described in Section 3.1 are essential to providing a robust representation of object locations in the environment. However, there are no mechanisms in the vision module that directly support spatial transformations, estimations, or calculations. While our discussion centers around processing spatial information obtained through visual perception, we accept that the mechanisms may generalize to other concepts, like volume in auditory perception, brightness in color perception, or intensity in taste, smell, and touch. Since ACT-R has only rudimentary auditory and vocal abilities (and no sense of taste, smell, or touch), these issues, in large part, cannot be addressed in the current architecture. They do, however, offer an interesting direction for future empirical and modeling research.

We propose that mechanisms for processing spatial information are instantiated within a specialized module in ACT-R. There are several key components of this

module, which we will examine in turn. The degree to which the different capabilities are actually separable from a cognitive or neuropsychological perspective needs to be carefully evaluated with additional research. In this section, we will attempt to differentiate them while simultaneously providing evidence that they are interdependent. These components of spatial ability include spatial transformations, magnitude estimations, and magnitude computations.

Spatial Transformations. Humans have the ability to maintain and manipulate mental images to perform a number of tasks and activities, from mental rotation [7], to image composition [60], to mental simulation [61], [62]. The ability to transform images and inspect the results is an essential component of spatial reasoning [49]. However, humans are capable of many different kinds of complex spatial image transformations, some of which depend heavily on knowledge and experience and/or are specific to particular object classes (e.g., compressing an accordion-like or spring-like object). Rather than address this broader class of complex transformations, we will initially model a small number of basic, but very frequently used transformations.

Perhaps the best-studied transformation to discuss is mental rotation. Although early research suggested that whole objects might be mentally rotated through intermediate states in a depictive representation [7], subsequent research argues for a more flexible process that can involve focusing on individual object parts, increasing their imagined size if necessary to make a fine discrimination, and changing their visualized position and orientation [6], [22]. Therefore we do not plan to model basic transformations such as size, position and orientation by directly moving the constituent points of an object across the visual icon. Rather, production rules will select a relevant object or object part as the focus of attention, resulting in its position being represented in the egocentric buffer. The production system will also select goal-relevant image transformation processes to alter the relevant slot values of the selected object/part. These transformations include translations, zooming, and mental rotation, which can be achieved by manipulating an image's *distance* and/or *bearing*, *size*, and *orientation* in the egocentric buffer, respectively. The vision module, using direct module-to-module links, recruits the spatial module to perform the operations.

The role of the spatial module in this case is to perform the requested transformations, producing alterations to the representation of the object in the visual icon. In many cases, this will be a complex, iterative process involving several objects or parts at different scales and in different locations. Often the next transformation subgoal will be determined only after the system inspects the results of the previous transformation, so image inspection processes will go hand-in-hand with spatial transformations. In addition, the decay properties of mental images, and perceptual refresh rates of visual stimuli will impact the size of the subgoals and other aspects of these transformation mechanisms. ACT-R already contains processes which control the inspection of simple visual information via the allocation of attention to locations and features represented in the icon.

Although the number of simple transformations that we will model is small, they can be combined using the process just described to create complex manipulations of mental images in the service of spatial cognition. An example of the usefulness of such transformations can be found in an analysis of the performance of expert meteorologists (e.g., [62], [63]). The comments of meteorologists upon viewing a

static display of weather patterns give evidence that they are generating weather predictions by imagining a complex series of transformations to the size, location, and orientation of regions affected by various meteorological events.

Magnitude Estimations. Just as the transformation component of the spatial module is closely linked with manipulating mental images, magnitude estimations are associated with encoding information using vision or mental imagery. According to Klatzky [33], estimates of egocentric distance and bearing are primitive values in egocentric frames of reference. In addition, however, humans are able to estimate these relations between arbitrary pairs of objects in the environment, which can be useful for many purposes, including navigation [64]. Of course, there is bias and error associated with these operations, but people are still able to achieve a relatively high degree of accuracy. These processes also appear to be involved in planning and executing motor movements, like reaching to grab a coffee cup on the desk in front of you (e.g., [65]). Estimating magnitudes is a basic function of the spatial module, and is another point at which we posit significant module-to-module communication. In reaching to pick up a coffee cup, for example, detailed spatial information is necessary to plan and execute the appropriate motor actions to grasp the cup. Moreover, people perform these actions precisely, without conscious awareness of the spatial information that is influencing their motor movements [66]. Thus, we believe that these interactions occur through cortical pathways outside the main production cycle in ACT-R.

Under this perspective, the production system of ACT-R is responsible for formulating the high-level action, like “pick up the coffee cup.” The motor module is then responsible for determining how to perform that action, which involves interaction with the spatial module to plan the details of the motor movements. Research by Brooks [67] suggests that spatial information processing is required to plan motor movements. It also suggests that there is an overlap between these mechanisms and the mechanisms required to generate, maintain, and inspect mental images, which were described above.

We believe that magnitude estimation is utilized by the vision module as well, when a new item is attended in the environment. When a shift of attention occurs, information about the distance and bearing of the object with respect to ACT-R’s location in the environment must be computed. We propose that these operations recruit the spatial module as well. It also may be the case that this component of the spatial module is involved in planning and executing eye movements to bring new items into the focus of attention (e.g., [68]). As noted above, these mechanisms may be applicable more broadly for computing magnitudes other than spatial quantities. However, consideration of those possibilities is beyond the scope of this proposal.

Lastly, the mechanisms of estimation can be engaged by the production system, through the spatial buffer. An explicit attempt to estimate a distance or bearing from one object to another in the environment would be an example of how central cognition may utilize these mechanisms. Such a request would result in a chunk, returned into the spatial buffer, which identifies the objects and the relationship between them. Such explicit requests form the basis of generating a *cognitive map* within this architecture. This set of mechanisms can also compute qualitative estimates of magnitudes, like *close*, *above*, *small*, and *far*. Some research has

suggested that qualitative (categorical) operations like this are localized in the left hemisphere, while quantitative (continuous) operations (e.g., distance/bearing estimates) are performed in the right hemisphere [69], [70].

Magnitude Computations. In some circumstances, like planning an attention shift or a motor movement, estimates of magnitudes can be useful in isolation. However, some of the most important functions of spatial cognition involve performing computations involving multiple magnitudes. There are a variety of computations that may be performed on magnitude information, including qualitative comparisons (e.g., $<$, $>$, $=$) and quantitative operations (e.g., $+$, $-$, $/$, $*$). Again, these different types of operations may be performed in different hemispheres in humans (c.f. [69], [70]). This is a sophisticated, and potentially extensive, set of operations to be performed on quantitative information.

We propose that these functions are computed using another set of mechanisms within the spatial module. There is also evidence that these operations are conducted on abstract representations of magnitude, rather than using information embedded in vision (e.g., distance and bearing) or other modality. Neuropsychological research has shown that the angular gyrus, in the posterior inferior parietal lobule, is implicated in the processing of spatial and numerical information (e.g., [71], [72]). We take these findings to suggest that quantitative information of this sort is represented in a common format for performing computations like those mentioned above. Thus, we propose that the outputs of estimation processes are in an abstract, propositional form. Comparisons and computations, then, are performed on this abstract representation. These requests are generated through central cognition, utilizing the spatial buffer mentioned above.

4 Spatial Competence in the Brain

Thus far, the discussion of the architecture for spatial competence has centered around the structure and mechanisms required to support spatial information processing within ACT-R. In this section, we present some information regarding the mapping of those structures and mechanisms to particular brain areas. Neuropsychological evidence concerning spatial abilities in humans is extensive. It has been shown that the parietal lobe is critical in processing spatial information, and a very large number of studies have attributed particular aspects of spatial cognition to particular portions of the parietal lobe and other portions of the cortex (c.f., [73]).

A comprehensive review of the neuropsychological evidence concerning spatial cognition is not presented here. What we do provide is an overview of the mapping of the spatial competence architecture to brain regions without considering the mapping of other components of ACT-R to the brain, which has been addressed elsewhere (e.g., [26]). Along the way, we cite important research that supports our position, but generally do not take time to examine all the perspectives. In addition, area delineations and hypothesized locations should be considered as approximate. There is a great deal of complexity in the human cortex, and we do not wish to suggest that cognitive functions are exclusively localized in the regions we suggest, nor do we

believe necessarily that these are the only functions performed by the various locations.

Figure 1 above contains our hypothesized assignment of components of our proposal to brain regions. We have placed the egocentric buffer in the posterior parietal lobe, within the superior parietal lobule. This follows research that has identified a distinction between a ventral *where* or *action* stream and a dorsal *what* stream [52], [66]. We view the egocentric buffer as representing the output of the ventral stream. Next, the environmental buffer, which encodes object location with respect to an exocentric frame of reference, is in the inferior portion of the lateral occipital gyrus. This area and nearby areas (including the parahippocampal cortex) have been associated, variously, with acquiring exocentric spatial information [74], representing the local visible environment [75], perceiving and encoding landmarks [76], [29], encoding ‘building stimuli’ [74], and encoding of ‘large objects’ [77]. All of these things can be seen to relate to identifying the location of an object with respect to an exocentric frame of reference based on what is visible in the surrounding environment.

We attribute to the hippocampus the role of encoding episodic information about visual experience (although it is plausible, even likely, that this incorporates other sensory modalities as well). This lines up closely with the description of the function of the hippocampus given by Kosslyn [22]. He states, “...the hippocampus may set up the neural equivalent of ‘pointers’, linking representations that are stored in different loci...” (p. 223). As noted earlier, others have posited other roles for the hippocampus in spatial cognition, particularly with regard to place cells and the *cognitive map*. Space limitations here prevent us from reviewing and commenting on the evidence relevant to this issue.

Spatial operations take place across the parietal lobe, as noted in Figure 1. However, we posit that the different functions we have identified for the spatial module can be localized to different parts of the parietal lobe. Still, even these more specific references represent substantial abstractions. For instance, the superior parietal lobule has been associated with visuospatial working memory operations [78], [79], [80], and we relate this region to the component of the spatial module that performs spatial transformations. The angular gyrus is active in spatial tasks generally [69], [71] and in tasks requiring calculations, particularly mathematics, more specifically [72], [81]. Thus, we associate the angular gyrus with performing magnitude computations. This conceptualization of the function of this area actually provides a unification across some of the different notions of the role of this portion of the cortex. Finally, proposals for the role of the supramarginal gyrus include directing spatial attention (e.g., [68]), mental imagery (e.g., [82]), and motor preparation [65]. All of these functions fit well with the role attributed to this area in our account, which is performing magnitude estimations. Additionally, these operations all rely on a representation of location (following [52]) to support action (as suggested by [66]). So once again, our theory provides a potential unification for seemingly disparate results.

Mental imagery is captured in Figure 1 in the connections between components of the vision module and the spatial module, which indicate processing links between brain regions. We have not yet associated all of these links with particular pathways in the brain, but there is evidence for at least some of them (c.f., [22]). This proposal

for the generation and manipulation of visual mental images lines up well with work by Kosslyn. Certainly, many of the details are missing in the current mapping of components of this account onto the brain, but the emerging view is consistent with what we know about mental imagery and neuropsychology.

Finally, the buffer for the spatial module resides in the frontal cortex. The frontal cortex is associated with high-level planning and goal maintenance activities. Additionally, dorso-lateral prefrontal cortex (DLPFC) shows enhanced activity in performing spatial tasks [83], [84]. We view this activity as stemming from the processing requirements of managing requests for spatial operations and harvesting the results of that processing. This anatomical relationship is similar to the proposed mapping of declarative memory to the brain in ACT-R, where the buffer is in ventro-lateral prefrontal cortex (VLPFC) and the actual storage of declarative information occurs in the temporal lobe and hippocampus [26].

In summary, we have accomplished a tentative mapping of spatial information processing structures and mechanisms to brain areas. The selective review we have presented illustrates that empirical and neuropsychological evidence generally supports the mapping we have developed. There is, as mentioned, a vast psychological literature relating to this topic, and there are many neuropsychological phenomena for which this mapping does not provide an account. As we implement the mechanisms, and validate the performance of the entire system against human empirical and neuropsychological data, we will use key findings in this literature to refine the mechanisms that are implemented to account for the processes that are occurring in the brain when humans perform spatial tasks.

5 Conclusion

We have described a set of mechanisms for human spatial competence. The architecture proposed is consonant with existing empirical and theoretical evidence regarding the capabilities and limitations of human spatial information processing, and is also consistent with current knowledge about the functional neuroanatomy of the brain. In addition, our account is integrated with the ACT-R cognitive architecture, which is a well-validated, quantitative theory of human cognition. As the scope of cognitive architectures expand, and as processing limitations of computer technology are overcome, it is critical that psychologically valid accounts of human spatial competence be implemented in cognitive architectures. Incorporating mechanisms for spatial competence will allow cognitive architectures, like ACT-R, to provide quantitative accounts of human performance in a wider range of task environments. This will be critical for achieving the goals of unified theories of cognition [47].

On the other hand, it is also vital that theories of human spatial competence incorporate mechanisms that account for capacities and limitations in human perceptual, cognitive, and motor performance. In any task, it is the interplay of the entire system that produces the behavior that can be observed. By linking our account to ACT-R, we can leverage the mechanisms of a well-validated theory of the human cognitive architecture. Mechanisms for spatial competence fill in a significant gap in ACT-R's capabilities, just as ACT-R provides detailed mechanisms for memory and

performance that link spatial competence to human cognition more broadly. Of course, the proposal we have described in this paper does not address every phenomenon in the literature on human spatial information processing, but it does provide an integrated framework that can be applied widely for understanding the capacities and limitations of human cognition in this area. As the structures and mechanisms are implemented, we will focus on the empirical, theoretical, and neuropsychological details, to ensure that our account is psychologically valid. For example, perhaps the processing mechanisms currently grouped within a single “spatial” module are better conceived as a set of 2, or even 3, separate modules that interact in spatial information processing. This has implications for capacities and processes, and these details will matter when the architecture is utilized to provide quantitative accounts of human performance. This and other issues will be addressed as we move forward. A critical point, however, is that a computational model, implemented within a cognitive architecture, is vital for tackling these issues at this level of detail. Thus, we are enthusiastic and optimistic about the potential for generating a unified, comprehensive account of how humans encode, store, and process spatial information.

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