Mental Models in Spatial Reasoning

Markus Knauff[1], Reinhold Rauh[1], Christoph Schlieder[2], & Gerhard Strube[1]

[1]University of Freiburg
Center for Cognitive Science
{knauff,reinhold,strube}@cognition.iig.uni-freiburg.de
[2]Technical University of Munich
Department of Computer Science
csc@tiki.informatik.tu-muenchen.de

Abstract. This chapter gives an overview of our ongoing experimental research in the *MeMoSpace* project, concerning the cognitive processes underlying human spatial reasoning. Our theoretical background is mental model theory, which conceives reasoning as a process in which mental models of the given information are constructed and inspected to solve a reasoning task. We first report some findings of our previous work and then two new experiments on spatial relational inference, which were conducted to investigate well-known effects from relational and syllogistic reasoning. (1) Continuity effect: n-term-series problems with continuous $(W\ r_1\ X,\ X\ r_2\ Y,\ Y\ r_3\ Z)$ and semi-continuous ($X\ r_2$ Y, $Y\ r_3\ Z,\ W\ r_1\ X$) premise order are easier than tasks with discontinuous order $(Y\ r_3\ Z,\ W\ r_1\ X,\ X\ r_2\ Y)$. (2) Figural bias: the order of terms in the premises *(X r Y, Y r Z* or *Y r X, Z r Y)* effects the order of terms in the conclusion *(X r Z* or *Z r X*). In the first experiment subjects made more errors and took more time to process the premises when in discontinuous order. In the second experiment subjects showed the general preference for the term order *Z r X* in the generated conclusions, modulated by a "figural bias": subjects used *X r Z* more often if the premise term order was *X r Y, Y r Z,* whereas *Z r X* was used most often for the premise term order *Y r X, Z r Y.* Results are discussed in the framework of mental model theory with special reference to computational models of spatial reasoning.

1 Introduction

In a large number of everyday contexts, people make extensive use of binary spatial relations which locate one object (LO = located object) with respect to another (RO = reference object). Examples of such relations are *"lies to the left"*, *"lies to the right"*, *"is in front of*" and so on. Furthermore, we are able to use such relations for making inferences, that is, we are able to infer relations not explicitly given from the ones we already know. If we know, for instance, that object X lies to the left of object *Y* and that *Y* is to the left of object *Z*, it is very easy for us to infer that *X* must be to the left of *Z*.

Such inferences based on binary spatial relations have long been studied in the psychology of thinking and have recently received increased attention in the literature on mental model theory of human reasoning (Johnson-Laird, 1983; Byrne & Johnson-Laird, 1989; Johnson-Laird & Byrne, 1991; Evans, Newstead & Byrne, 1993).

The general scheme of an important class of tasks studied in the psychology of reasoning are the so called *n-term-series problems*, in which subjects have to find a conclusion on the basis of given premises. In the special case of a spatial *three-term series problem (3ts-problem)*, two spatial relational terms $X\ r_1\ Y$ and $Y\ r_2\ Z$ are given as premises (Johnson-Laird, 1972). The goal is to find a conclusion $X\ r_3\ Z$ that is consistent with the premises. In a *four-term series problem (4ts-problem)*, three premises $W\ r_1\ X$, $X\ r_2\ Y$ and $Y\ r_3$ Z, are given, and three relations not explicitly given, namely $W\ r_4$Y, $X\ r_5$ Z and $W\ r_6\ Z$, can be inferred. However, there are slight differences depending on which inference paradigm is being applied. Two such paradigms are commonly found in the literature on reasoning. The first we will call an *inference verification task*. The second, the *active inference task*, can be broken down into two different cases, *active general inference* and *active particular inference*. To make the difference explicit, we introduce the notation $\{\varphi_1,\varphi_2\} \triangleright \varphi_3$ to denote the fact that the conclusion φ_3 is consistent with the premises φ_1 and φ_2. The two paradigms can be written as follows:

(1) *inference verification:* does $\{\varphi_1,\varphi_2\} \triangleright \varphi_3$ hold?

(2a) *active general inference:* find <u>all</u> φ_3 such that $\{\varphi_1,\varphi_2\} \triangleright \varphi_3$.

(2b) *active particular inference:* find <u>some</u> φ_3 such that $\{\varphi_1,\varphi_2\} \triangleright \varphi_3$.

There are two main theories that attempt to explain the underlying mental processes of such inferences. The first is called *theory of mental proof* and goes back to the idea that the human mind contains something like a mental logic consisting of formal inference rules (Rips, 1994). According to this theory, language-like and context-independent formal rules of inference are represented in the human mind, and inference tasks are solved by applying these rules to the given premises. Rips (1994) characterized the main idea as follows:

> *"... reasoning consists in the application of mental inference rules to the premises and conclusion of an argument. The sequence of applied rules forms a mental proof or derivation of the conclusion from the premises, where these implicit proofs are analogous to the explicit proofs of elementary logic". (Rips, 1994, p. 40)*

The key idea of the theory is clearly a repertoire of inference rules represented in human long-term-memory (LTM). These rules can be used to solve inference tasks by transferring them into working memory (WM) and applying them to the given premises, which are also represented there. For this reason, the given premises must be kept separate in the mind throughout the whole reasoning process, which means that no integrated or unified representation of the given information is generated. The premises as well as the inference rules are represented as separate entities in WM.

Here lies the main difference between this and the second approach, *mental model theory.* Since this second approach seems to be empirically more successful, and is the theoretical background of our project, we will briefly review the essential points of mental model theory of spatial relational inference in the following section. However, it is important for the reader to keep in mind that although we restrict ourselves here to the spatial domain, the theory also accounts for other types of reasoning.

1.1 Spatial Inference According to Mental Model Theory

In general, the key idea of *mental model theory* is that people translate an external situation of the real world into a *mental model* and use this representation to solve given inference tasks. In other words, a mental model is a representation of objects and relations ("structure") in working memory that constitutes a model (in the usual logical sense) of the premises given in the reasoning task. According to this view, spatial reasoning does not rely primarily on syntactic operations like in the rule-based approaches, but rather on the construction and manipulation of mental models (Johnson-Laird & Byrne, 1991). The common denominator of all mental model accounts is the conception of reasoning as a process in which, at first, unified mental representations of the given premises are generated and then, due to the fact that this information can be ambiguous, alternative models of the premises are sequentially generated and inspected. This process can be broken down into three separate phases, which are often called the comprehension, description and validation phases (Johnson-Laird & Byrne, 1991). In our project, we use the terms *construction, inspection* and *variation phase* in order to clarify the character and function of these phases.

In the *construction phase* reasoners use their general knowledge and knowledge about the semantics of spatial expressions to construct an internal model of the "*state of affairs*" that the premises describe. This is the stage of the reasoning process in which the given premises are integrated into an unified mental model. According to the theory, only this mental model needs to be kept in memory, i.e. the premises may be forgotten. It is important to point out that numerous spatial descriptions are vague and that often we can find more than one possible model that is consistent with the given premises. For this reason, we have to distinguish *determinate* tasks in which only a single model can be constructed from *indeterminate* tasks that make multiple models possible. The influence of this difference on the difficulty of reasoning tasks is one of our main research topics and we will return to this point later.

In the *inspection phase*, a parsimonious description of the mental model is constructed, including a preliminary conclusion. In other words, the mental model is inspected to find out relations which are not explicitly given. This phase was called the description phase by Johnson-Laird and Byrne (1991) because they conceived the preliminary conclusion as a kind of description of the model: *"This description should assert something new that is not explicitly stated in the premises"* (Johnson-Laird & Byrne, 1991, p. 35).

In the *variation phase*, people try to find alternative models of the premises in which the conclusion is false. If they cannot find such a model, the conclusion must be true. If they find a contradiction, they return to the first stage – and so on until all possible models are tested (Johnson-Laird & Byrne, 1991). For this reason the variation phase could be viewed as an iteration of the first two phases in which alternative models are generated and inspected in turn.

Also characteristic of the mental model theory is the concentration on particular questions, namely (1) why some inference tasks are harder to solve than others, and (2) how the inference process starts and which of the possible models reasoners generate first.

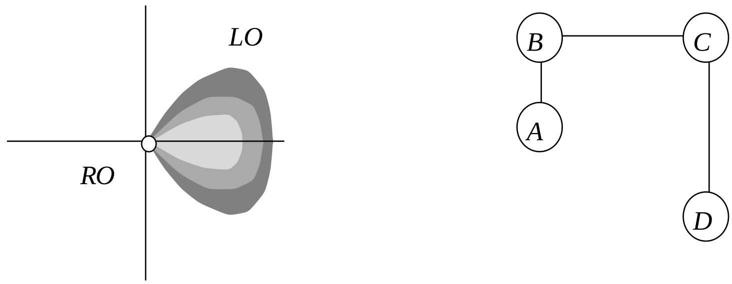
Mental model theory answers the first question quite simply: mental models are entities represented in working memory, which has a very limited capacity. Due to this fact, an inference task becomes more difficult with an increasing number of possible models that the reasoner has to keep in mind. According to this hypothesis, inference tasks with multiple models must be harder than those with single models, which means that they will take longer to solve and will be more likely to result in errors.

The second question is discussed in the literature under the keywords “initial” or “preferred models” and was investigated in our project extensively. In general, we can say that in multiple model cases, the sequence of generated models is not random. On the contrary, the construction of a first mental model seems to be a general cognitive process that works the same way for most people. We will later discuss this question in more detail.

Before we summarize our most relevant empirical results together with some findings reported in the literature, and report two new experiments, we must say a few words about the material used in our experiments.

2 The Material: Spatial Relations with an Unambiguous Semantic

When investigating spatial relational inference, cognitive psychologists usually present natural language expressions (such as “*right-of*”, “*left-of*”, “*in-front-of*”, “*behind*”) in spatial descriptions to their subjects (Byrne & Johnson-Laird, 1989; Ehrlich & Johnson-Laird, 1982; Mani & Johnson-Laird, 1982). However, as illustrated in Fig. 1, the semantics of these natural language expressions and their underlying relational concepts are far from being clear. As illustrated in the lefthand picture, numerous empirical results (in particular those concerning ratings of acceptability) have shown that the semantics of spatial descriptions such as *“lies to the right”* are very ambiguous and fuzzy (see for example Gapp, 1997; Knauff, 1997). It is not clear how to deal with such problems when investigating spatial inferences involving these natural language descriptions. The problem is more clearly demonstrated in the righthand picture. It is easy to see in this picture that the relation of the natural language expressions to the diagram is far from being clear and makes an assessment of the mental models generated by the subjects very difficult.



Ratings of acceptability for "LO lies to the right of RO"

Is this configuration a model of "D is before A"?

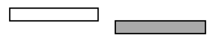
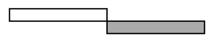
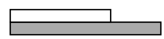
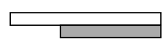
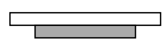
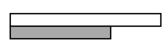
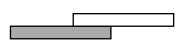
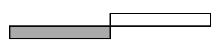
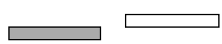
Fig. 1. The two examples illustrate problems with spatial expressions. As shown in the lefthand figure, the semantics of "right " is vague and fuzzy. The righthand figure demonstrates such a problem in the context of spatial inference tasks.

To avoid these difficulties in our experiments, we use the set of thirteen qualitative relations introduced by Allen (1983). In AI research on spatial reasoning the interval relations are commonly used as a representational device and found their way into several applications, e.g. in geographic information systems. There is one main reason for preferring this set of relations over natural language expressions in our experiments: one can formulate a model-theoretic semantic of the relations that allows the exact determination of what counts as a model and what does not (e.g., Nebel & Bürckert, 1995; Schlieder, 1995). This is because the relations are *jointly exhaustive* and *pairwise disjoint* (JEPD), i.e. exactly one relation holds between any two intervals. It is easy to see that these 13 relations can be used to express any qualitative relationship that can hold between two (one-dimensional) objects in an inference task.

Later it should be possible to generalize the results from our experiments in order to understand inferences on the bases of a hitherto unknown natural set of relations. In this sense, Allen's set of relations can be used as a means to study the properties of the inference processes in the spatial domain.

The set consists of the following 13 relations: *before (<)* and its converse *after (>)*, *meets (m)* and *met by (mi)*, *overlaps (o)* and *overlapped by (oi)*, *finishes (f)* and *finished by (fi)*, *during (d)* and *contains (di)*, *starts (s)* and *started by (si)*, and *equal (=)*, which has no converse. Table 1 gives pictorial examples for these relations, natural language expressions for the spatial domain, and the ordering of startpoints and endpoints as the basis for the model-theoretical foundation. In the following, we will refer to a specific point ordering as an "ordinal model", because it ignores the metrical properties of the spatial description and is based solely on ordering information of the startpoints and endpoints of the intervals.

Table 1. The 13 qualitative interval relations, associated natural language expressions, one graphical realization, and ordering of startpoints and endpoints (adapted and augmented according to Allen, 1983).

Symbol	Natural language description	Graphical realization	Point ordering (s=start, e=endpoint)
X < Y	*X lies to the left of Y*		$s_X < e_X < s_Y < e_Y$
X m Y	*X touches Y at the left*		$s_X < e_X = s_Y < e_Y$
X o Y	*X overlaps Y from the left*		$s_X < s_Y < e_X < e_Y$
X s Y	*X lies left-justified in Y*		$s_Y = s_X < e_X < e_Y$
X d Y	*X is completely in Y*		$s_Y < s_X < e_X < e_Y$
X f Y	*X lies right-justified in Y*		$s_Y < s_X < e_X = e_Y$
X = Y	*X equals Y*		$s_X = s_Y < e_Y = e_X$
X fi Y	*X contains Y right-justified*		$s_X < s_Y < e_Y = e_X$
X di Y	*X surrounds Y*		$s_X < s_Y < e_Y < e_X$
X si Y	*X contains Y left-justified*		$s_X = s_Y < e_Y < e_X$
X oi Y	*X overlaps Y from the right*		$s_Y < s_X < e_Y < e_X$
X miY	*X touches Y at the right*		$s_Y < e_Y = s_X < e_X$
X > Y	*X lies to the right of Y*		$s_Y < e_Y < s_X < e_X$

Combining two relations r_i and r_j gives the composition $c(r_i, r_j)$ that specifies the possible relationships between an interval X and *Z* given the qualitative relationship between X and Y, and Y and Z. For instance, given that *X meets Y* and *Y is during Z* then the following relations between *X* and *Z* are possible: *X overlaps Z* or *X is during* Z or *X starts with Z*. Since Allen's theory contains thirteen relations, there are 144 compositions $c\ (r_1\ r_2)$, when omitting the trivial "=" relation. They are presented in Table 2. As can be seen in the example mentioned above, there are compositions (exactly 72 of the 144) that have multiple solutions. They are presented in the table as shaded cells, whereas the white cells are single model cases. From these compositions it is easy to construct inference tasks that are known in the psychology of reasoning as 3ts-problems (e.g. Johnson-Laird, 1972).

Table 2. Composition table for the 12 qualitative relations (omitting the trivial relation "=") introduced by Allen (1983). (shaded cells = multiple model cases; white cells = single model cases)**.**

	<	m	o	fi	di	si	s	d	f	oi	mi	>
<	<	<	<	<	<	<	<	<, m, o, s, d	<, m, o, s, d	<, m, o, s, d	<, m, o, s, d	<, ..., >[*]
m	<	<	<	<	<	m	m	o, s, d	o, s, d	o, s, d	fi, =, f	di,si, oi, mi, >
o	<	<	<, m, o	<, m, o	<, m, o, fi, di	o, fi, di	o	o, s, d	o, s, d	o,fi, di,si, =,s,d, f,oi	di,si, oi,	di,si, oi, mi,>
fi	<	m	o	fi	di	di	o	o, s, d	fi, =, f	di, si, oi	di, si, oi	di,si, oi, mi,>
di	<, m, o, fi, di	o, fi, di	o, fi, di	di	di	di	o, fi, di	o, fi, di,si, =,s, d, f, oi	di, si, oi	di, si, oi	di, si, oi	di,si, oi, mi,>
si	<, m, o, fi, di	o, fi, di	o, fi, di	di	di	si	si, =, s	d, f, oi	oi	oi	mi	>
s	<	<	<, m, o	<, m, o	<, m, o, fi, di	si, =, s	s	d	d	d, f, oi	mi	>
d	<	<	<, m, o, s, d	<, m, o, s, d	<, ..., >	d, f, oi, mi,>	d	d	d	d, f, oi, mi,>	>	>
f	<	m	o, s, d	fi, =, f	di,si, oi, mi,>	oi, mi, >	d	d	f	oi, mi,>	>	>
oi	<, m, o, fi, di	o, fi, di	o, fi, di,si, =,s, d, f, oi	di, si, oi	di, si, oi, mi, >	oi, mi, >	d, f, oi	d, f, oi	oi	oi, mi, >	>	>
mi	<, m, o, fi, di	si, =, s	d, f, oi	mi	>	>	d, f, oi	d, f, oi	mi	>	>	>
>	<, ..., >[†]	d, f, oi, mi,>	d, f, oi, mi,>	>	>	>	d, f, oi, mi,>	d, f, oi, mi,>	>	>	>	>

*. All 13 relations are possible.

3 Empirical Evidence and Previous Work

In the following section we will give a brief overview of our previous work, which used Allen's calculus as a basis, and discuss some empirical results reported in the literature of mental model theory. The sections about the "order of premises" and the "figural effect" describe two effects that are very often found in experiments concerned with other kinds of deductive reasoning and which can be largely accounted for by the assumptions of mental model theory. The question as to whether we could also find such effects in the spatial domain motivated us to perform the two experiments reported afterwards in detail.

3.1 Premise Integration in Spatial Relational Inference

As outlined above, rule theories assume that inferences are drawn on the basis of the linguistic-semantic representation of the premises, whereas mental model theory states that the premises are integrated into a unified representation–the mental model. Evidence for an integrated representation was gained (i) indirectly and (ii) mainly in the field of transitive inference (Maybery, Bain & Halford, 1986; Johnson-Laird & Byrne, 1991).

Therefore, we conducted an experiment using the *active particular inference paradigm* (2b in section 1) that was aimed to test for premise integration in spatial relational reasoning tasks in a direct manner (for details see Rauh & Schlieder, 1997). Subjects read referentially continuous, indeterminate 4ts-problems and were asked to construct one possible relationship between each implicit pair of intervals separately, namely between the first and the third, the second and the fourth, and the first and the fourth interval. Giving subjects the opportunity to provide answers to the three implicit relationships separately, a distinction could be made between correct answer triples and model-consistent answer triples. Since the latter make up a subclass of the former, they have the additional property that the three answers together with the premises were consistent with an integrated representation of the premises, whereas the correct, but non model-consistent answer triples needed to be consistent only with the premises. Materials were selected to minimize the ratio of model-consistent answers to correct answer triples, in order to result in a strong test for premise integration. The main result of this experiment was that nearly all correct answer triples were also model-consistent, which is strong evidence that people constructed one integrated representation of the premises and scanned it for the implicit informative relationships.

3.2 Model Construction: Preferred Mental Models

Within the mental model framework with its three phases of inference, we concentrated further on the phase of construction of an initial model of the premises. As a general theory of human reasoning, mental model theory ought to explain the construction of mental models from the premises as a serial process that always produces the same first mental model.

In a further experiment, also using the *active particular inference paradigm* (2b in section 1; see: Knauff, Rauh & Schlieder, 1995 for details) we tested the assumption of the existence of generally preferred mental models: subjects had to read 3ts-problems and give one possible relationship between the first and the third interval. We were able to show that a significant majority of subjects were in agreement with respect to the given answer for all of the 72 indeterminate (multiple models) problems. This suggests that the construction of an initial mental model is a general cognitive process that seems to work the same way for most people. The preferred models, with respect to the composition table (shown in Table 2), are presented in Table 3.

Table 3. Empirical model preferences (Knauff, Rauh & Schlieder 1995).

	<	m	o	fi	di	si	s	d	f	oi	mi	>
<	<	<	<	<	<	<	<	<,d	o	o	o	=
m	<	<	<	<	<	m	m	o	o	o	=	oi
o	<	<	<	<	m	o	o	o	o,d	=	oi	>
fi	<	m	o	fi	di	di	o	d	=,f	oi	oi	>
di	<	o	o	di	di	di	o	=	oi	oi	oi	>
si	<	o	o	di	di	si	=	d	oi	oi	mi	>
s	<	<	o	o	fi	si	s	d	d	oi	mi	>
d	<	<	o	o	=	oi	d	d	d	oi	>	>
f	<	m	o	fi	di,oi	oi	d	d	f	oi	>	>
oi	<	o	=	oi	mi	mi	d	oi	oi	>	>	>
mi	<	si	oi	mi	>	>	oi	oi	mi	>	>	>
>	=	oi	oi	>	>	>	oi	>	>	>	>	>

3.3 On the Causal Influence of Preferred Mental Models

Based on the fact that the initially constructed mental model is the first one that is available in working memory, it follows that this will favor certain inferences before others. We tested this prediction in an experiment using the *verification task paradigm* (1 in section 1), where subjects first read referentially continuous indeterminate 3ts-problems and then had to verify a presented relationship between the first and the third interval (see Rauh, Schlieder & Knauff, 1997, for details). As shown in Fig. 2 the results corroborated our prediction in two ways: relationships that conformed to the preferred mental model in the study of Knauff et al. (1995) were (1) verified faster than other possible relationships, and (2) they were also more often correctly verified than other possible relationships.

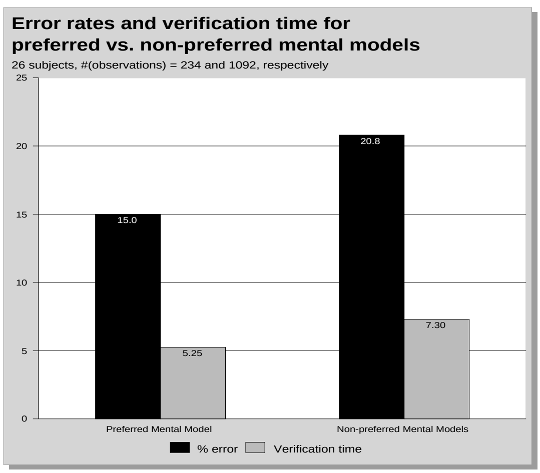


Fig. 2. Percentage of errors and verification latencies [in sec.] for 3ts-problems combined with relations that correspond to the preferred mental models as opposed to the same problems combined with other valid relations.

3.4 Symmetry Properties of Preferred Mental Models

Investigating the process of model construction in more detail, we aimed at characterizing abstract properties of this process for a cognitively adequate algorithmic reconstruction. With recourse to formal studies of Allen's calculus (Ligozat, 1990), it is possible to investigate (i) whether the model construction process works in the same manner from left-to-right and right-to-left, and (ii) whether processing an interval relation is dependent on what has been already processed (context-dependent processing of spatial relationships). Rauh and Schlieder (1997) devised an experiment in which related pairs of referentially continuous indeterminate 4ts-problem were presented in a generation experiment (2b in section 1). For each original 4ts-problem there was a twin problem that differed only with respect to orientation and an additional twin problem that differed from the original only with respect to the transposition (see Fig. 3). The main conclusion of this study was that the model construction process works in the same manner from left-to-right or right-to-left, but is context-sensitive, i.e., the processing of spatial relationships is dependent on what already has been processed. These two general properties, *symmetry of reorientation* and *asymmetry of transposition*, rule out whole classes of possible cognitive modelings and provide restrictions that a cognitively adequate algorithmic reconstruction must take into account.

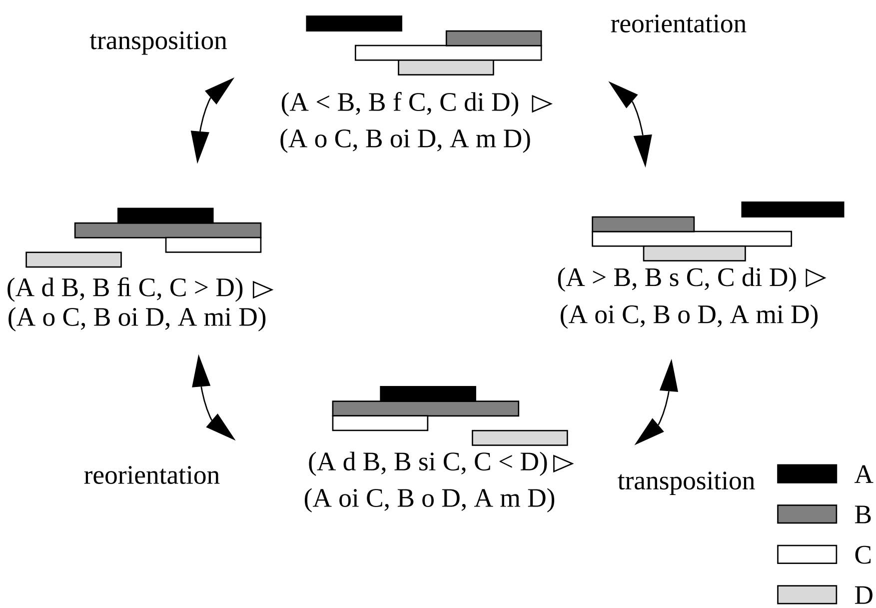


Fig. 3. Symmetry transformations on Allen-based 4ts-problems constituting an *orbit* of 4 inference tasks.

In summary, we obtained empirical evidence that (1) people construct an integrated representation of the premises, (2) that they generally come up with the same integrated representation–the preferred mental model, (3) that these preferred mental models have causal effects, because they facilitate certain inferences and suppress others, and (4) that the model construction process has the two properties of working in the same manner from left to right and right to left and of being context-sensitive.

3.5 The Order of Premises Effect

Further evidence that people construct integrated representations of the given premises in the sense of a mental model has been found through the investigation of premise order. The reported result is often called in the literature "continuity effect" or "order of premises effect" (Evans, Newstead & Byrne, 1993).

Ehrlich and Johnson-Laird (1982), for instance, gave subjects relational 4ts-problems and the three premises $W\ r_1\ X$, $X\ r_2\ Y$, $Y\ r_3\ Z$ were presented in continuous *(*$W\ r_1\ X$, $X\ r_2\ Y$, $Y\ r_3\ Z$*)*, semi-continuous *(*$X\ r_2\ Y$, $Y\ r_3\ Z$, $W\ r_1\ X$*)*, and discontinuous order *(*$Y\ r_3\ Z$, $W\ r_1\ X$, $X\ r_2\ Y$*)*. Subjects had to infer only the conclusion $X\ r_4\ Z$. The dependent measures were the error rates (and premise processing times) for each kind of premise order.

The results support the prediction of mental model theory that continuous order (37% errors) is easier than discontinuous order (60% errors) and there is no significant difference between continuous and semi-continuous (39% errors) tasks.

Mental model theory explains these results as an effect of the difficulty of integrating the information from the premises. In the continuous and semi-continuous orders, it is possible to integrate the information of the first two premises into one model at the outset, whereas when they are presented with the discontinuous order subjects must wait for the third premise in order to integrate the information in the premises into a unified representation. Before they get this information they have to temporarily store the information from the first and second premise separately, making the task much harder.

Experiment 1 below was conducted to investigate the effect of premise order in spatial relational inference through the application of Allen's interval relations.

3.6 The Figural Effect

When investigating the effect of premise order an obvious question is whether there is a similar effect for the order of objects (terms) inside the premises. This has been done extensively in the area of syllogistic reasoning and researchers have come up with an extremely reliable and very robust effect that is called the "figural effect" or "figural bias" (Hunter, 1957; De Soto, London & Handel, 1965; Trabasso, Riley & Wilson, 1975). We explain this effect according to an experiment on relational inference by Johnson-Laird and Bara (1984). They asked subjects for a possible conclusion (1 in section 1) for the following types of problems:

Type 1:	***Type 2:***
X is related to Y	***Y is related to X***
Y is related to Z	***Z is related to Y***

The result was, that in problems of Type 1 subjects tend to spontaneously generate more conclusions in the form "*X is related to Z*" than the other correct conclusion *"Z is related to X"*, whereas they tend to generate more conclusions in the form *"Z is related to X"* for problems of Type 2. According to the rule-based, mental proof theory, the surface features of the premises determine the figural effect (Rips, 1994). However, Johnson-Laird and Bara (1984) explained the "figural effect" according to mental model theory. They assumed that the integration of the premises in working memory is more difficult in Type 2 problems because of the need to bring the *Y* term into the middle. According to this view, the construction of a mental model is easier for premises that have the repeated term as first term in the next premise. In this case, the information of the given premises can be integrated immediately and no cognitive resources are needed for mental operations that bring the middle term into the middle.

There are many good reasons to be sceptical as to whether a figural effect can also be found in spatial relational inference tasks. In particular, the syntactic structure of spatial tasks - without quantifiers - are very similar to each other and there are only a few different surface features.

Experiment 2 below was conducted to find out whether a figural effect can be found in the spatial domain as well.

4 Experiment 1: Order of Premises

As in the experiments reported above, this computer-aided experiment was separated into three blocks: a *definition*, a *learning*, and an *inference phase*. The reasons for the procedure are discussed extensively in Knauff, Rauh and Schlieder (1995). The main idea was to distinguish between conceptual and inferential aspects of Allen's calculus and to refer the obtained results to the pure inference process, holding constant the conceptual aspects.

Subjects

Thirtysix paid students (18 female, 18 male) of the University of Freiburg, ranging in age from 21 to 33 years.

Method and Procedure

In the *definition phase*, subjects read descriptions of the locations of a red and a blue interval using the 13 qualitative relations (in German). Each verbal description was presented with a short commentary about the location of the beginnings and endings of the two intervals and a picture with a red and blue interval that matched the description.

The *learning phase* consisted of blocks of trials, where subjects were presented with the one-sentence description of the red and blue interval. They then had to determine the startpoints and endpoints of a red and a blue interval using mouse clicks. After confirmation of her/his final choices, the subject was told whether her/his choices were correct or false. If they were false, additional information about the correct answer was given. Trials were presented in blocks of all 13 relations in randomized order. The learning criterion for one relation was accomplished if the subject gave correct answers in 3 consecutive blocks of the corresponding relation. The learning phase stopped as soon as the last remaining relation reached the learning criterion. Subjects needed 15 to 30 minutes to accomplish the learning phase.

In the *inference phase*, subjects had to solve 12 spatial *4ts-problems* in the *active particular inference paradigm* (2b in section 1), and the premises $W\ r_1\ X$, $X\ r_2\ Y$, and $Y\ r_3$ Z were presented in continuous ($W\ r_1\ X$, $X\ r_2\ Y$, $Y\ r_3$ Z), semi-continuous ($X\ r_2\ Y$, $Y\ r_3$ Z, $W\ r_1$ X) and discontinuous ($Y\ r_3\ Z$, $W\ r_1\ X$, $X\ r_2\ Y$) order. They were selected on the basis of our first 4ts-experiment reported above, thus the number of correct answers given by the subjects were relatively high and each of the 12 relations were presented in the first premise exactly once. According to the separated-stages paradigm (Potts & Scholz, 1975), premises were presented successively in a self-paced manner, each on an extra screen.

Afterwards, subjects had to specify the three conclusions, namely the implicit relations $W\ r_4$ Y, $X\ r_5$ Z and $W\ r_6\ Z$, each on an extra screen, by choosing the startpoints and endpoints of the intervals in lightly colored rectangular regions, as they had done in the learning phase. To avoid the effects of presentation order we systematically varied the color of the intervals and the order of conclusions asked for on the separate screens. This made the tasks relatively difficult, since subjects not only had to specify the relations but also to remember the combination of colors in each premise.

The three instances of each of the 12 4ts-problems (12x3=36 tasks) were compiled in different blocks, and there was also one practice block in the beginning consisting of 6 other simple 4ts-problems. The sequence of experimental blocks was counterbalanced across subjects according to a sequentially counterbalanced Latin square. The experiment took approximately 1.5 hours.

4.1 Results

All 36 subjects successfully passed the learning phase, and all data collected in the inference phase could be further analyzed. Individual performance showed considerable variation, ranging from 44% to 95% correct answers.

As shown in Fig. 4, the results corroborated our prediction in two ways: (1) there was no significant difference in the percent of errors between continuous (39.7%) and semi-continuous (40.1%) premise order, but (2) both were significantly easier than the discontinuous order which lead to 50.0% errors on average [$\chi^2_{(1)} = 9.643$, $p < .001$; $\chi^2_{(1)} = 8.864$, $p < .002$.].

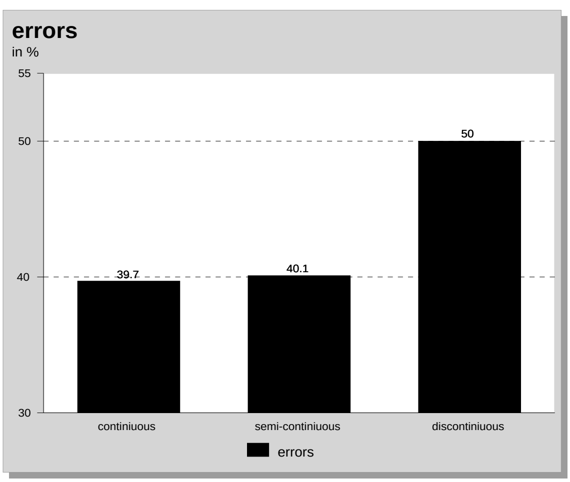


Fig. 4. Error rates for continuous, semi-continuous and discontinuous premise order in the 4ts-problems using Allen's interval relations.

Another important finding is reported in Table 4: the data on premise processing times support the assumption of mental model theory that a discontinuous premise order will increase the processing time for the third premise, because information from all premises must be integrated at this point.

Table 4. Premise processing times for the first, second and third premise in the tasks with continuous, semi-continuous and discontinuous premise order.

	Premise processing time in sec.		
premise order	premise 1	premise 2	premise 3
continuous	13.0	11.2	10.9
semi-cont.	13.6	11.0	14.4
discontinuous	12.4	13.9	19.5

Reliable differences can be found in the processing times of the third premise between continuous and semi-continuous order [$F(1,35) = 37.61$, $p < .001$], semi-continuous and discontinuous order [$F(1,35) = 40.44$, $p < .001$], and continuous and discontinuous order [$F(1,35) = 74.87$, $p < .001$]. For the second premise the differences between continuous and discontinuous order [$F(1,35) = 17.63$, $p < .001$], and semi-continuous and discontinuous order [$F(1,35) = 22.89$, $p < .001$] are significant. All other differences, in particular in the first premise, and the difference between continuous and semi-continuous order in the second premise, are not reliable.

4.2 Discussion

The experiment was conducted to investigate the continuity effect in the spatial domain with the aid of the interval relations. The error rates as well as the premise processing times showed a strong continuity effect. Subjects made more errors in tasks with discontinuous premise order than in continuous and semi-continuous order and it took more time to process the third premise in the discontinuous condition. These results can be seen as evidence for the most important assumption of mental model theory, namely that the information of the premises is integrated in a unified representation–the mental model. With this background, the result can be explained as an effect of the difficulty of integrating the information from the premises. Only in the continuous and semi-continuous order, is it possible to integrate the premises immediately into one unified representation, whereas in the discontinuous order the information from the first and second premise must be kept temporarily separated (may be in a language-like propositional form or as separate models) in working memory until the third premise is given.

This assumption is supported by the premise processing times as well, which have shown that it took much more time to process the third premise in the discontinuous order. Again, these results are compatible with the assumption that subjects build an integrated representation of the given premises. In fact, the processing time for the third premise in the discontinuous premise order must be longer, because at this point in the model construction process subjects get the first opportunity to integrate the first two premises.

5 Experiment 2: Order of Terms

As mentioned above, the figural effect is a very robust finding in the area of syllogistic reasoning (Johnson-Laird & Bara, 1984; Johnson-Laird & Steedman, 1978) that was also found in relational reasoning (Johnson-Laird & Bara, 1984, Exp. 2). For the spatial domain, however, the figural effect has not yet been investigated systematically. For this reason, the following experiment is an explorative one designed to determine whether there is the same figural effect in spatial relational inference, and whether the order of terms effects the preferred mental model.

Subjects

Twentyfour paid students (12 female, 12 male) of the University of Freiburg, ranging in age from 20 to 33 years.

Material and Procedure

The computer-aided experiment was again separated into the three phases. The definition phase and the learning phase were conducted as in Experiment I. In the *inference phase* subjects had to solve spatial *3ts-problems* (plus 10 practice trials) in the *active particular inference paradigm* (2b in section 1).

Of the 144 possible 3ts-problems, we selected 32 indeterminate task (i.e., multiple model problems) that showed the highest degree of preference from our preferred mental models experiment reported in Knauff, Rauh and Schlieder (1995). For each task we constructed "twin" tasks, which use the inverse relation but describe the same spatial relation between the three intervals. For example, the spatial arrangement of *"X lies to the left of Y"* and *"Y lies to the left of Z"* is identical to *"Y lies to the right of X"* and *"Z lies to the right of Y"*.

As shown in Table 5, based on the location of the terms, we constructed tasks of four different types (4x32=128 3ts-problems). The complement lines in the table denote the fact that the inverse relation was used. With respect to the terminology of research on syllogistic reasoning the "types" can also be called "figures". In all four types ***Y*** is the middle term, which occurs in both premises of the problem but on different locations. The conclusions connect the two end terms ***X*** and ***Z***, which occur in the premises at different locations as well.

Table 5. The 3ts-problems of experiment 2 were constructed in four different types, by changing the term orders and using the inverse interval relations.

type	premise 1	premise 2	possible conclusions
1	$X\, r_1\, Y$	$Y\, r_2\, Z$	$X\, r_3\, Z$ *or* $Z\, \bar{r}_3\, X$
2	$Y\, \bar{r}_1\, X$	$Y\, r_2\, Z$	
3	$X\, r_1\, Y$	$Z\, \bar{r}_2\, Y$	
4	$Y\, \bar{r}_1\, X$	$Z\, \bar{r}_2\, Y$	

In each trial, after reading the premises, subjects first had to decide which interval to use to begin the one-sentence description of the conclusion (in German). This was done by pressing associated keys on the keyboard, namely <B> for *"The blue interval* ...", <R> for *"The red interval* ..." and <G> for *"The green interval* ...". Afterwards a new screen was shown, where this phrase and the second part of the sentence was displayed automatically. This was possible because the middle term could not be used in the conclusion. If, for example, the green interval was the middle term of the task, and the subject had pressed the key <R> initially, the two phrases *"The red interval* ..." and "... *the blue interval"* were displayed. Between these, a list of all 13 interval relations were displayed (in randomized order), and the subject could choose one of them with the cursor.

5.1 Results

As in our previous experiment, all 24 subjects successfully passed the learning phase. Individual performance in this (easier) experiment ranged from 43% to 98% correct answers.

The most important result is concerned with the term orders chosen in the conclusions. It can be predicted from the results of Johnson-Laird and Bara (1984) that subjects tend to choose the order $\boldsymbol{X\, r_3\, Z}$ (abbreviated in the following as "***X* – *Z***") in the conclusion more often than the reverse order $\boldsymbol{Z\, \bar{r}_3\, X}$ (abbreviated as "***Z* – *X***"). This assumption is not supported by our results: 62.8% of all conclusions given by the subjects were in the order ***Z* – *X***.

Independently from this result, we analyzed how often the conclusions ***X* – *Z*** and ***Z* – *X*** were used for the four types of premise term orders. As shown in Fig. 5, the term order ***X* – *Z*** was used for ***X* – *Y*, *Y* – *Z*** (44.3%) more often than for ***Y* – *X*, *Y* – *Z*** (38.7%)**,** ***X*– *Y*, *Z* – *Y*** (39.6%), and ***Y* – *X*, *Z* – *Y*** (31.1%). The difference between ***Y* – *X*, *Y* – *Z*** and ***X* – *Y*, *Z* – *Y*** is not reliable [$\chi^2_{(1)} = 0.134$, $p > .71$], whereas the other differences are significant and show a figural bias: The conclusion ***X* – *Z*** was used more often for the premise order ***X* – *Y*, *Y* – *Z*** than for ***Y* – *X*, *Y* – *Z*** [$\chi^2_{(1)} = 4.959$, $p < .015$], ***X* – *Y*, *Z* – *Y*** [$\chi^2_{(1)} = 3.465$, $p < .035$] and ***Y* – *X*, *Z* – *Y*** [$\chi^2_{(1)} = 28.259$, $p < .001$] and for the premise order ***Y* – *X*, *Y*– *Z*** more often than for ***Y* – *X*, *Z* – *Y*** [$\chi^2_{(1)} = 9.640$, $p < .001$]. The difference between ***Y* – *X*, *Z* – *Y*** and ***X* – *Y*, *Z* – *Y*** is also reliable [$\chi^2_{(1)} = 12.036$, $p < .001$].

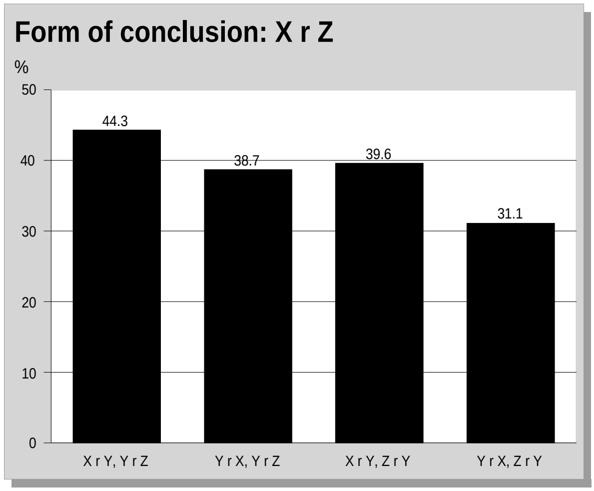


Fig. 5. The effect of term order in the premises on the form of conclusions. The figure shows the distribution of X r Z conclusions [in %] for 32 problems in each of the four types.

The next question is which type of problem is more difficult. In general, with an relatively low error rate of 16.4% the subjects performed the tasks surprisingly well. In Fig. 6 the error rates for the four term orders are depicted. Only the difference between ***Y – X, Y – Z*** and ***X– Y, Z – Y*** is reliable [$\chi^2_{(1)} = 7.107$, $p < .05$]. We have also analyzed the premise processing times, but found no significant differences.

As mentioned above, in an earlier experiment we have found preferred mental models for problems with multiple solutions (see Table 3). We now look at the solutions of our 32 indeterminate problems and compare the conclusions with these preferences. Two results are important: (1) the preferences we found in the experiment were independent from the order of terms. Only in one of the 32 tasks did the preferences differ in the four term orders; (2) in all cases we found strong preferences, the majority of which (24 out of 32, or 75%) were identical to those found in our previous investigation (Knauff & al., 1995).

Finally we computed an item- and subject analyses and found slight differences for both factors. First, the subjects‘ tendency to prefer the conclusion term order ***Z – X*** was not equally distributed [$\chi^2_{(23)} = 363.440$, $p < .001$]. In fact ***X – Z*** conclusions were chosen by five of our 24 subjects in more than 50% of the problems. Second, the item analyses also have shown differences for the tasks with respect to the distribution of ***X – Z*** and ***Z – X*** conclusions [$\chi^2_{(31)} = 54.842$, $p < .005$].

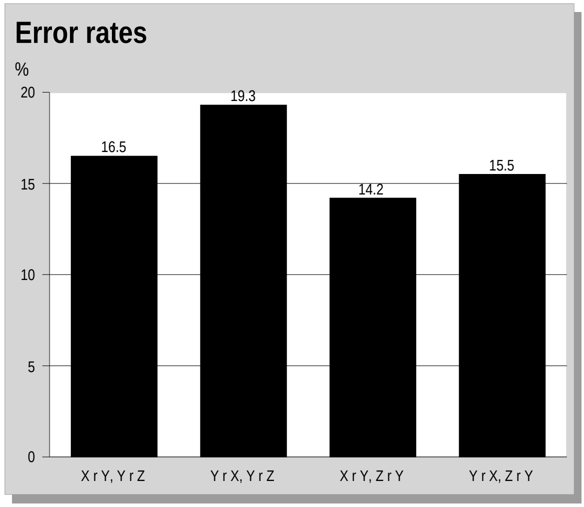


Fig. 6. Error rates [in %] for the four types of term order.

5.2 Discussion

As mentioned above, Experiment 2 were aimed more at proving the existence of some effects found in other domains of reasoning, than on testing predictions from mental model theory of spatial relational inference. We found (i) contrary to the results of Johnson-Laird and Bara (1984) no general bias towards ***X* – *Z*** conclusions, and (ii) in accordance with Johnson-Laird and Bara a figural effect, i.e. the figure ***X* – *Y*, *Y* – *Z*** favored ***X* – *Z*** conclusions, whereas the figure ***Y* – *X*, *Z* – *Y*** favored ***Z* – *X*** conclusions. Additionally, (iii) we found significant differences for the ***X* – *Z*** preference between subjects, and (iv) minor, but still reliable differences for the ***X* – *Z*** preference between different tasks.

The first contradictory result of the ***Z* – *X*** preference could be attributed either to the spatial domain or to the property of the used relation in the Johnson-Laird and Bara study; they used the relation “is related to” denoting kinship, a relation that has the property of symmetry in contrast to the normally used material in relational reasoning studies (“better than”, “taller than”, ...) and also in contrast to our qualitative relations that do not have the property of symmetry. The effect of abstract properties of relations like symmetry, asymmetry, or anti-symmetry, and the effect of the domain of relations (spatial v. non-spatial) on the preference of ***X* – *Z*** conclusions has to be determined in future experiments. At least, the overall effect of a ***Z* – *X*** preference can be explained by a cognitive process that inspects the mental model by means of a spatial focus and is sensitive with respect to the outcome of the model construction. This explanation is sketched in the general discussion below.

The next result was that the type of preferred mental model seems to be independent of the order of terms in the premises, since in nearly all the *3ts problems* the same relation between the end terms was chosen for all four figures per task. Also, the stability of preferred mental models determined in the study of Knauff et al. (1995) was not perfect, but within the range of variability found in a replication of the latter experiment in Kuß, Rauh and Strube (1996).

That some subjects differed in the overall preference for ***Z* – *X*** conclusions and that some tasks favored ***Z* – *X*** conclusions more than others may have to do with different inspection strategies (for example re-focussing before inspection of the mental model or not) and preferred direction of inspection of mental models (for example preferred inspection direction from left-to-right as practiced heavily in the everyday activity of reading). Subsequent detailed analysis of tasks and further experiments investigating this factor systematically have to be carried out in the future.

6 General Discussion

We reported two experiments investigating the "continuity effect" and the "figural bias" in spatial relational inference tasks. Taken together, our findings support an account of the inference process following mental model theory. In the first experiment we found clear evidence for premise integration and showed that discontinuous premise order is much harder than semi-continuous and continuous order. The result is easy to explain on the basis of the difficulty of integrating the information of the premises. This explanation is clearly supported by the premise processing times. In general the outcome of the experiment, together with numerous similar findings in other areas of inference, leads to a homogeneous image of what makes one inference task more difficult than others.

On the other hand, the results of the second experiment leave us with some open questions. The main idea of the experiment was to investigate another factor possibly effecting spatial relational inference in a similar fashion to the way that premise order does. The results of this experiment were surprising, particularly in one point. Contrary to the results of Johnson-Laird and Bara (1984), we found no general bias towards ***X* – *Z*** conclusions. In fact, subjects tend to generate conclusions of the form ***Z* – *X***. At first glance, this result seems to be counterintuitive, but may have a plausible explanation: the overall effect of a ***Z* – *X*** preference can be explained–in agreement with the assumptions of mental model theory–by a cognitive process that inspects the mental model by means of a spatial focus (see for example: De Vooght & Vandierendonck, in press). In this case, after model construction the focus should be positioned on the last end term of the *3ts-problem*, namely ***Z***. If this is the starting point of the scanning process it is plausible that the first term in the conclusion is ***Z*** and not ***X***, because for ***X* – *Z*** conclusions the focus must be shifted back to the term ***X*** before the scanning process starts. In contrast, for ***Z* – *X*** conclusions these time consuming refocussing processes are unnecessary. This seems to be a plausible explanation for the preference of ***Z* – *X*** conclusions in our experiment. In addition to this, the cognitive modeling reported in the next section, gives support to this explanation.

However, the most important result of the experiment is that the model construction process seems to be independent of the term order in the premises. In 31 of the 32 problems the same (preferred) mental model was chosen for all four types of term order.

Taken together, both experiments give us important hints into the processes of model construction and model inspection. The model construction process seems to be sensitive with respect to the premise order and widely independent of the term order. In particular the difficulty of premise integration under discontinuous circumstances seems to be a strong argument for an incremental model construction process and supports the assumptions of mental model theory. The importance of term order emerges when the constructed model is inspected. The model inspection process can be explained as a scanning process, in which a focus is shifted over the mental model. This process seems to be sensitive to the last position of the focus, which is a result of the model construction process (see below). However, further investigations are needed to decide whether model construction and model inspection interact with respect to the positioning of the spatial focus.

7 A Computational Approach to Spatial Reasoning with Allen's Interval Relations

Schlieder (1995) proposed a computational theory of the processes we investigated. The relevant data for which the computational theory claims to account come in particular from the experiments on preferred mental models and symmetry transformations described briefly above and in detail in Knauff, Rauh, and Schlieder (1995) and Rauh and Schlieder (1997).

In the following section, we offer a brief look at the main idea of this computational model. In addition, we hope that this sketch gives an impression why the preference of ***Z – X*** conclusions in experiment 2 is plausible. However, the major motivation for formulating a computational theory is the expectation that some general principles guide the model construction process and thereby explain why the preferred mental models arise. In addition, this approach helps to decide what type of spatial information is relevant for the model construction process.

For the tasks investigated in our project, the model construction process could involve either metrical information or ordering information. The former is the more constraining (i.e., stronger) and the latter the less constraining (i.e., weaker) type of spatial information. Following the principle of representational parsimony, our account is based on ordering information alone: the model of two premises $X\ r_1\ Y$ and $Y\ r_2\ Z$ is an (ordinal) *point ordering representation* that represents only the linear order of the *startpoints* and *endpoints* of the intervals. From the many different ways in which a linear order of a startpoint and endpoint can be represented, we choose the most parsimonious, namely to represent only the direct succession and the identification of points.

The modeling also accounts for the asymmetries of our experimental data by making the outcome of the model construction process dependent on the status in which the model was left by the previous premise integration. This status consists of the described point ordering plus a focus position. The model of ***X m Y*** for instance is represented as shown in Fig. 7.

$$e_{min} < s_X < e_X = s_Y \underset{\text{focus}}{\boxed{<}} e_Y < s_{max}$$

Fig. 7. A model of X *m* Y.

In this account, the inference is modeled as a scanning process over this representation that is realized as a shift of the focus. A further assumption is that the scanning process is directed in one of two possible directions and that the *model construction* should require only a minimum number of changes in scanning direction. Our idea of explaining the ***Z – X*** preference comes in exactly at this point. Our modeling is mostly concerned with the *model construction* process, but it is also plausible to postulate a similar shift of focus in the *model inspection* phase. Recapping: the focus should be positioned on the last end term ***Z*** after the model construction process. If the shift of focus for the *model inspection* requires as well only a minimum number of changes in scanning direction, the avoidance of a re-focussing to the first term ***X*** after the model construction is not surprising. For model inspection, it is much easier to use this point as starting point of the scanning process. In this case the scanning process starts with the end term ***Z*** and not with ***X*** and we get the term order ***Z – X*** in the generated conclusion.

However, for a brief assessment of the computational theory, we will return to the empirical results of our earlier experiments, as the modeling only claims to account for the data from the experiments on preferred mental models and symmetry transformations. We compared the empirical model preferences with the predictions of the computational theory. To sum up, assuming that the model construction process essentially works in the same manner in a left-to-right direction as in a right-to-left direction, only 2 out of the 72 preferences cannot be reproduced by the computational theory. To our mind, the cognitive modeling can therefore be considered as descriptively adequate. However, the experimental results with respect to the figural effect–which have not yet been taken into account–may help us to come up with an increasingly adequate cognitive modeling of spatial relational inference. Apart from this it may help us to compare our approach to a second account in explaining our preferences, which was developed in another group of the DFG special program. This approach goes back to the idea of metrical prototypes as the basis of model construction and comes up with some other predictions (Berendt, 1996).

8 The Conceptual Adequacy of Allen's Interval Relations

So far we have only reported experimental results which are concerned with the inferential aspects of Allen's calculus, and the reader will remember that our experiments all started with a learning phase to control the conceptual aspects of the calculus. However, one might ask whether Allen's interval relations can also be used to describe a part of human conceptual knowledge. In a previous paper on the cognitive adequacy of Allen's calculus we addressed this question by differentiating between two kinds of cognitive adequacy, namely *conceptual* and *inferential* cognitive adequacy (Knauff, Rauh, & Schlieder, 1995). Our understanding of *inferential cognitive adequacy* can be claimed if and only if the reasoning mechanism of the calculus is structurally similar to the way people reason about space. This question was the main topic of this paper. Independently from this, *conceptual cognitive adequac*y can be claimed if and only if empirical evidence supports the assumption that a system of relations is a model of people's conceptual knowledge of spatial relationships.

To answer this question for Allen's interval relations, a number of experiments were conducted and are reported in Knauff (1997; for a short overview see: Knauff, Rauh, Schlieder & Strube, 1997). In addition to several experiments in which subjects gave acceptability ratings for spatial arrangements with respect to Allen‘s relations or generated spatial arrangements for themselves, two memory experiments were conducted.

In a recognition experiment, for instance, subjects learned spatial arrangements with respect to Allen's relations (targets) and were later presented with them together with instances of other relations as distractors. The task was to find out the earlier learned arrangements (target) and the question was whether or not subjects are able to differentiate all 13 possible relations. We also predicted that the task must become more difficult with an increase in spatial similarity between target and distractor. In a recall experiment, subjects had to generate (draw) the previously learned spatial relations after getting a cue-stimulus.

Taken together, the most important result of these experiments was that subjects could remember the learned arrangements very well (recognition: 92%; recall: 60%) and are able to distinguish all 13 relations without learning anything about them before. To our mind, this result is a strong argument that some parts of human conceptual knowledge about ordinal spatial arrangements can be described with respect to Allen's interval relations. For our project this has the consequence that we are not working with absolutely artificial material but can say that it seems to be conceptually adequate in at least some important aspects (for details see: Knauff, 1997).

To conclude this section we wish to point out that in a new paper we report our research in cooperation with another group from the DFG special program that is concerned with a very similar question translated to the domain of topological knowledge (Knauff, Rauh & Renz, 1997; see also the article of Renz and Nebel in this book).

9 Conclusions and Future Work

Together with the conclusions of our earlier experiments the reported results show a good explanatory coherence within the framework of mental model theory, and challenge other theories of reasoning like approaches based on formal rules of inference. In particular the findings of Experiment 1 support the assumption that spatial relational inference is based on the construction of unified representations. However, further evidence will be needed before a detailed modeling of the inference process is possible. Experiment 2 leaves it an open question as to why the preferred term order in the conclusions is modulated by the term order of the premises. A series of ensuing experiments will be concerned with this point and the interaction of model construction and model inspection. Furthermore, we will investigate whether mental models represent only ordinal spatial information as assumed in the presented computational model, or contain metrical information as well. At present, we are performing dual-tasks experiments, in which the second task is “visual” or “spatial” as distinguished for example in Logie (1995) or Kosslyn (1994). The results will probably give some hints as to the kind of information represented in spatial mental models.

10 Acknowledgments

The reported research was supported by grant *Str 301/5-1* (MeMoSpace) in the research program “Spatial cognition” of the National Research Foundation (Deutsche Forschungsgemeinschaft, DFG). We would like to thank Karin Banholzer, Thomas Kuß and Kornél Markó for their assistance in several parts of the investigations and Bernhard Nebel, Nick Ketley, Judas Robinson, Karl-Friedrich Wender and an anonymous reviewer for helpful comments on an earlier draft of this paper.

References

Allen, J. F. (1983). Maintaining knowledge about temporal intervals. *Communications of the ACM, 26,* 832-843.

Berendt, B. (1996). Explaining preferred mental models in Allen inferences with a metrical model of imagery. *Proceedings of the Eighteenth Annual Conference of the Cognitive Science Society.* (pp. 489-494). Mahwah, NJ: Lawrence Erlbaum Associates.

Byrne, R. M. J., & Johnson-Laird, P. N. (1989). Spatial reasoning. *Journal of Memory and Language, 28,* 564-575.

De Soto, L. B., London, M. & Handel, L. S. (1965). Social reasoning and spatial para-logic. *Journal of Personality and Social Psychology,* 2, 513-521.

De Vooght, G., & Vandierendonck, A. (in press). Spatial mental models in linear reasoning. *Kognitionswissenschaft (Special issue “Räumliche mentale Modelle / Spatial mental models”).*

Ehrlich, K., & Johnson-Laird, P.N. (1982). Spatial descriptions and referential continuity. *Journal of Verbal Learning and Verbal Behavior, 21,* 296-306.

Evans, J. St. B. T., Newstead, S. E., & Byrne, R. M. J. (1993). *Human reasoning. The psychology of deduction*. Hove (UK): Lawrence Erlbaum Associates.

Freksa, C. (1991). Qualitative spatial reasoning. In D. M. Mark & A. U. Frank (Eds.), *Cognitive and linguistic aspects of geographic space* (pp. 361-372). Dordrecht: Kluwer Academic Publishers.

Gapp, K.-P. (1997). *Objektlokalisation [Object Localisation].* Wiesbaden: Deutscher Universitäts-Verlag.

Hunter, I. M. L. (1957). The solving of three-term series problems. *British Journal of Psychology, 48,* 286-298.

Johnson-Laird, P. N. (1972). The three-term series problem. *Cognition, 1,* 58-82.

Johnson-Laird, P. N. (1983). *Mental models. Towards a cognitive science of language, inference, and consciousness*. Cambridge, MA: Harvard University Press.

Johnson-Laird, P. N., & Bara, B. G. (1984). Syllogistic reasoning. *Cognition*, 16, 1-62.

Johnson-Laird, P. N., & Byrne, R.M.J. (1991). *Deduction.* Hove: Lawrence Erlbaum Associates.

Johnson-Laird, P. N., & Steedman, M. (1978). The psychology of syllogisms. *Cognitive Psychology, 10*, 64-99.

Knauff, M. (1997). *Räumliches Wissen und Gedächtnis [Spatial knowledge and memory]*. Wiesbaden: Deutscher Universitäts-Verlag

Knauff, M., Rauh, R., & Renz, J. (1997). A cognitive assessment of topological spatial relations: Results from an empirical investigation. In S. C. Hirtle & A. U. Frank (Eds.), *Spatial information theory. A theoretical basis for GIS.* Proceedings of COSIT '97 (pp.193-206). New York: Springer.

Knauff, M., Rauh, R., & Schlieder, C. (1995). Preferred mental models in qualitative spatial reasoning: A cognitive assessment of Allen's calculus. In *Proceedings of the Seventeenth Annual Conference of the Cognitive Science Society* (pp. 200-205). Mahwah, NJ: Lawrence Erlbaum Associates.

Knauff, M. Rauh, R., Schlieder, C., & Strube, G. (1997). Analogizität und Perspektive in räumlichen mentalen Modellen [Analog representation and perspective in spatial mental models]. In C. Umbach, M. Grabski & R. Hörnig (Hrsg.). *Perspektive in Sprache und Raum* (pp. 35-60). Wiesbaden: Deutscher Universitäts-Verlag.

Knauff, M., Rauh, R., Schlieder, C., & Strube, G. (1998). Continuity effect and figural bias in spatial relational inference. *Submitted to the Twentieth Annual Conference of the Cognitive Science Society*, Madison-Wisconsin.

Kosslyn, S. M. (1994). *Image and brain.* Cambridge, MA: MIT Press.

Ligozat, G. (1990). Weak representations of interval algebras. *Proceedings of the Eighth National Conference on Artificial Intelligence* (Vol. 2, pp. 715-720). Menlo Park, CA: AAAI Press / MIT Press.

Logie, R. H. (1995). *Visuo-spatial working memory.* Hove: Lawrence Erlbaum.

Maybery, M.T., Bain, J.D., & Halford, G. S. (1986). Information-processing demands of transitive inference. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 12,* 600-613.

Nebel, B., & Bürckert, H.J. (1995). Reasoning about temporal relations: A maximal tractable subclass of Allen's interval algebra. *Communication of the ACM, 42,* 43-66.

Potts, G. R., & Scholz, K.W. (1975). The internal representation of a three-term series problem. *Journal of Verbal Learning and Verbal Behavior, 14,* 439-452.

Rauh, R., & Schlieder, C. (1997). Symmetries of model construction in spatial relational inference. In *Proceedings of the Nineteenth Annual Conference of the Cognitive Science Society* (pp. 638-643). Mahwah, NJ: Lawrence Erlbaum Associates.

Rauh, R., Schlieder, C., & Knauff, M. (1997). Präferierte mentale Modelle beim räumlich-relationalen Schließen: Empirie und kognitive Modellierung [Preferred mental models in spatial relational inference: Empirical results and cognitive modeling]. *Kognitionswissenschaft, 6, 21-34.*

Rips, L. J. (1994). *The psychology of proof.* Cambridge, MA: MIT Press.

Schlieder, C. (1995). *The construction of preferred mental models in reasoning with the interval relations* (Tech. Rep. 5/95). Freiburg: Institut für Informatik und Gesellschaft der Albert-Ludwigs-Universität Freiburg (appears also in: C. Habel & al. (Eds.), Mental models in discourse comprehension and reasoning).

Trabasso, T., Riley, C. A., & Wilson, E. G. (1975). The representation of linear orders and spatial strategies in reasoning. A developmenal study. In R. J. Falmagne (Ed.), *Reasoning: Representation and Processes.* New York: Wiley.